# Determining The Uncertainty Of Ionospheric Corrections For Users In Wide Area Augmentation Systems


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## ABSTRACT

Broadcast messages of the FAA's Wide Area Augmentation System (WAAS) include the grid ionosphere vertical error (GIVE). The GPS signal delay due to the ionosphere is estimated in real-time at each of the WAAS grid points; GIVE values bound the estimation error. In the overall WAAS architecture, GIVE is not used directly but as an intermediate step in computations of slant range error bounds for users. Therefore, WAAS performance should improve if the GIVE algorithm takes the user computation into account. We have developed a heuristic model of the systematic errors incurred when user slant delays are inferred from vertical ionosphere delays at fixed grid points. These systematic errors depend on real-time ionospheric conditions that vary widely over time-scales of hours to years (i.e. quiet/storm, day/night, winter/summer, solar-maximum/solar-minimum). Our heuristic error model

uses real-time WAAS reference station data as input to monitor ionospheric conditions and adjust the computed GIVE values accordingly. For example, it is well known that errors converting vertical delays to slant depend at least partially on horizontal gradients of ionosphere delay. Therefore, spatial gradients in the ionosphere are continuously estimated in real-time, and used to vary the GIVE in a regionally specific manner. We will discuss critical features of the algorithm that improve performance at the borders of the continental US and during major ionospheric storms.

## 1. INTRODUCTION

The FAA's Wide Area Augmentation System broadcasts corrections to the civilian GPS signal, improving the accuracy and availability of satellite-based aircraft navigation. To achieve a high degree of navigation integrity, the user is also provided the uncertainty of the corrections themselves. As long as the transmitted uncertainties bound the true errors of the user's range corrections, the user is protected from misleading navigation information.

The ionosphere range corrections are transmitted as vertical delays at a discrete set of fixed ionospheric grid points (IGP). A real-time mapping algorithm updates the grid delays every few minutes, based on ionospheric measurements from the WAAS reference stations (WRS). Users interpolate the grid delays to arbitrary locations within each grid cell and apply an obliquity factor for slant propagation paths [RTCA Special Committee, 1996]. The integrity information is also transmitted for each IGP as a vertical delay uncertainty: the grid ionosphere vertical error or GIVE.

We report on a GIVE algorithm that has been developed for WAAS (after phase 1) in partnership with the prime contractor Raytheon Systems. As originally proposed, WAAS phase 2/3 is meant to provide precision approach availability over 100% of the Continental US (CONUS). Analysis has shown [Conker *et al.*, 1997; Ahmadi *et al.*, 1997] that the Phase 1 algorithm provides adequate safety and availability over a central "egg-shaped" portion of the CONUS but does not provide adequate availability near the borders. The GIVE computation is accompanied by a stringent monitoring algorithm that ensures sufficient data are available to make a reliable uncertainty estimate. For Phase 1, delay measurements must be located in three of the four quadrants surrounding an IGP or the IGP is declared unusable or "not monitored". Since the WRS are placed exclusively within CONUS, it is difficult to "surround" IGPs near the borders (a simple consequence of geometry).

In the next section we describe an algorithm for computing the grid ionosphere vertical error (GIVE) that is being developed to satisfy the availability requirements of Phase 2/3 WAAS (100% of CONUS). We present GIVE validation results, using real-time measurements from the FAA's National Satellite Test Bed (NSTB) system and independent data from GPS receivers distributed throughout the CONUS. We then discuss the results including algorithm performance during a severe storm, and we conclude with a description of further testing and development work.

## 2. ALGORITHM RATIONALE

Generally, error bound estimation is a self-consistency or post-fit residual calculation, comparing the WRS measurements and the derived grid point delays [Conker *et al.*, 1997; Chao *et al.*, 1996]. This naturally requires that sufficient data are available in some area surrounding an IGP. Hence an additional grid point monitoring algorithm is used to ensure the computed GIVE estimates are reliable. To achieve the required availability over all of CONUS, this monitoring algorithm requires modification.

The purpose of the monitoring condition is to guarantee the reliability of the error bounds. If there were always sufficient data to compute reliable uncertainty estimates, then an appropriate GIVE would be sufficient to guarantee navigation integrity. Raytheon has proposed a method for including all of CONUS in the high-availability region by changing how grid point monitoring is accomplished.

In the new approach, the GIVE itself depends on the distribution of WRS ionospheric pierce points (IPP), reducing the need for a completely separate monitoring algorithm. The GIVE increases in the absence of WRS measurements surrounding an IGP. If the increase is sufficient to bound the correction error, the user is protected from hazardous navigation errors.

The new GIVE is based on the covariance of the parameters estimated in the ionosphere correction computation [Mannucci *et al.*, 1995]. The Kalman filter provides information on the variance of the estimated delay at the WAAS ionospheric grid points. This "formal error" reflects the quantity, geographical distribution and latency of data surrounding an IGP. Variances are available for all IGPs in real-time, and smoothly increase outward from the coverage boundaries, because of the more limited density of WAAS reference station (WRS) measurements near the borders of CONUS.

Although the use of formal error provides for a smooth and reasonable variation of the GIVE from the interior outwards, it has limitations as the sole source of an error bound for users [*Chao et al.*, 1996b]. The filter estimating the corrections is subject to mismodeling, particularly during ionospheric disturbances, so the true estimation errors may not always be reflected in the formal error. Another limitation of formal error is that it applies only to vertical delay uncertainties. Users must convert the vertical delays and error bounds to slant raypaths that are not collocated with the IGPs. Even if an IGP delay is known exactly, the slant conversion will introduce error. GIVE is therefore increased when ionospheric conditions or user geometries are unfavorable for the computation of slant range corrections. *The GIVE is explicitly designed to bound user slant range error.*

## 3. DESCRIPTION OF GIVE TERMS

The GIVE algorithm consists of three terms combined in a root-sum-square fashion. Each term is meant to account for a different component of error in the user's slant range correction. The GIVE calculation is performed sequentially for each IGP, using data in the four quadrants surrounding each IGP. As we've stated, the design increases GIVE if ionospheric conditions local to an IGP are such that users might incur larger errors.

The first GIVE term is meant to bound the error of the vertical IGP delay estimate, and is based on the covariance estimate from the ionosphere estimation filter. The second term increases the GIVE because of unmodeled ionospheric spatial decorrelation. The third term increases the GIVE in the presence of ionospheric delay gradients. These are known to decrease the accuracy of vertical-to-slant path scaling factors such as the MOPS obliquity factor. The three GIVE terms are more precisely defined below.

### 3.1 FORMAL ERROR TERM

The variance of the ionosphere delay estimate at the IGP, as computed in the Kalman filter, is the basis for the formal error GIVE term. The variance is scaled by a goodness-of-fit metric computed from the difference between the WRS measurements and the fitted ionospheric delay map. The fit residuals will generally increase under adverse ionospheric conditions. The formal error term for GIVE is:

$$GIVE_{FE} = \alpha 3.2\sqrt{\chi_2}\sigma_{FE} \qquad (1)$$

where $\sigma_{FE}$ is the root-variance of the vertical delay estimate (formal error) at the IGP, and $\alpha$ is an overall scale factor. $\chi_2$ is the goodness-of-fit metric "chi-squared" (Press *et al.*, 1992) computed using the $N$ available measurements within the four quadrants surrounding the IGP. It is defined as:

$$\chi_2 = \frac{1}{N}\sum_{i=1}^{N}\frac{(Fit_i - Data_i)^2}{\sigma_i^2} \qquad (2)$$

where $Fit_i$ is the slant delay at the measurement IPP computed using the WAAS wide-area correction algorithm, and $Data_i$ are the WRS delay data obtained over the last five minutes. $\sigma_i^2$ is the given measurement noise variance. GIVE is meant to bound 99.9% of the errors; the factor 3.2 assumes that the IGP delay errors follow a normal distribution with standard deviation $\sigma_{FE}$. The $\alpha$ factor is greater than one to account for non-gaussian statistics.

### 3.2 SPATIAL DECORRELATION TERM

A possibly significant contribution to user range error is unmodeled spatial variability or decorrelation of the ionosphere. We expect in a general sense that user pierce points at large distances from the nearest IGP would have larger errors associated with the ionospheric correction. We would also expect increased errors if the reference measurements are far from the IGP location. We want GIVE to increase with one or both of these distances.

The second GIVE term is meant to estimate the effects of ionospheric spatial decorrelation that are not well modeled by the fixed WAAS ionospheric grid. The most challenging situation for a user is when her IPP is far from the nearest WRS measurement. Conversely, if a user IPP is near WRS measurements, the goodness-of-fit metric described earlier ought to account for the mismodeling. Since decorrelation between two ionospheric locations increases with the distance between them [Klobuchar *et al.*, 1995], this term should increase with the distance between the user pierce point and the nearest WRS measurement.

The GIVE decorrelation term is defined as follows:

$$GIVE_{DEC} = \beta V_{MAX} D_{MAX} \qquad (3)$$

where $D_{MAX}$ is the maximum possible distance between the pierce point of a user and the closest WRS measurement. Computing $D_{MAX}$ assumes the user pierce points could be anywhere within the four quadrants surrounding an IGP[1]. $V_{MAX}$ is the largest vertical delay value somewhere within the four quadrants surrounding the IGP, as computed from the vertical correction map. It is included because we expect that ionospheric decorrelation effects increase with the overall level of ionospheric delay. The scale factor $\beta$ is chosen based on estimates of ionospheric decorrelation derived from experience with the real-time systems operating at JPL. The value used in the results reported here is shown in Table 1 below. Both the constant and the functional form of this term may be revised somewhat as the result of a planned study.

### 3.3 GRADIENT TERM

The MOPS obliquity factor used to scale vertical delays to slant corrections is based on an ionospheric shell model. In actuality, the ionospheric plasma is distributed vertically over several hundred kilometers. The shell model approximation introduces errors that depend at least in part on the delay gradients in the vicinity of the pierce point [Chavin, 1996a; Prag, 1996; Tsedilina and

Weitsman, 1992; Klobuchar *et al.*, 1993). The third GIVE term includes this dependence explicitly as follows:

$$GIVE_{GRAD} = \gamma\nabla_{MAX} \quad (4)$$

where $\nabla_{MAX}$ is an estimate of the maximum delay gradient surrounding the IGP. The ionospheric correction map is evaluated sequentially at the IGP location and a series of locations surrounding the IGP in several directions. $\nabla_{MAX}$ is determined from the largest delay change between the central and radial points (gradient = delay change/distance). The distance used in this computation is approximately the distance between WAAS grid points (5°). The factor $\gamma$ is based on estimates of vertical-to-slant conversion errors derived from model simulations conducted at JPL. The literature is also a useful resource [Tsedilina and Weitsman, 1992; Conker and El-Arini, 1998]. The value used currently is included in Table 1.

**Table 1**

| GIVE Parameter | Value | Comments |
|---|---|---|
| $\alpha$ | 1.4 | Exceeds one because error are non-gaussian |
| $\beta$ | $1.0\times10^{-7}$/meter | Increase vertical error by 5% at distances of 500 km |
| $\gamma$ | $1.0\times10^{6}$ meter | Increase vertical error by 0.5 meters when gradient is .5 cm/km (max value during a day) |

### 3.4 COMBINED GIVE

After the individual GIVE terms are computed for each IGP, the total GIVE is computed by combining the terms in a root-sum-square sense:

$$GIVE = \sqrt{GIVE_{FE}^2 + GIVE_{DEC}^2 + GIVE_{GRAD}^2} \quad (5)$$

Finally, the GIVE is rounded up to the discrete set of values available for transmission [RTCA Special Committee, 1996]. For the tests reported here, GIVE is updated every five minutes, the same rate as the IGP delays. Portions of the computation could be repeated more often for the purpose of setting alarms.

## 4. RESULTS

We present validation results for the new GIVE, using the system shown in Figure 1. Real-time data from the FAA's National Satellite Test Bed (NSTB) network are used to compute the ionosphere correction, instead of data from the WAAS reference stations. Slant observations from an independent GPS network in the CONUS are used as ground truth to validate the GIVE. Such data are freely available in an archive at JPL or at Scripps Oceanographic Institute and other data centers. A map showing the locations of NSTB and independent receivers is shown in Figure 2.

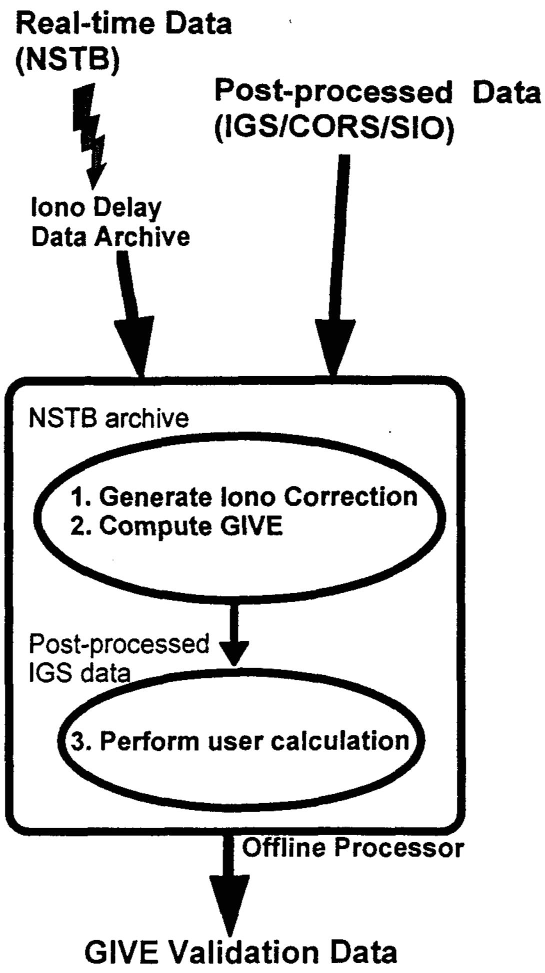


**Figure 1.** System for validating GIVE.

The real-time ionospheric data are transferred hourly to a local archive for storage, and processed in software that computes the WAAS ionosphere correction and GIVE every five minutes The independent GPS data are downloaded from external archives and used to generate ionospheric delays in a post-processed mode for maximum accuracy. For a validation run, the independent slant observations are compared to the ionospheric corrections to assess the GIVE performance over a variety of US locations.

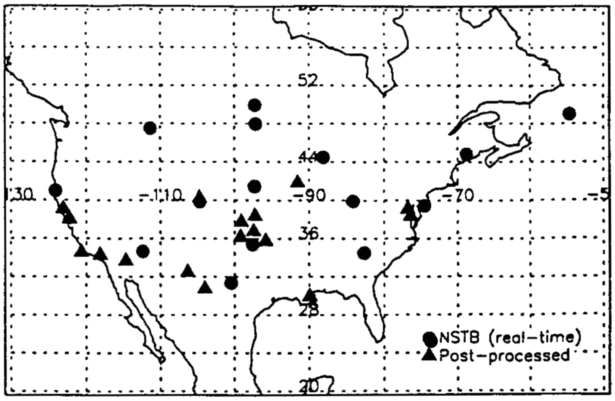


**Figure 2**. Map showing locations of NSTB (●) and independent validation receivers (▲).

GIVE validation results for one quiet and one moderately disturbed day are shown in Figures 3 and 4 (August 25th and August 6th 1998, respectively). These are scatter plots of the magnitude of the ionosphere correction error, actual versus computed, after scaling to vertical using the MOPS obliquity factor. The actual correction error is the difference between the WAAS ionosphere calibration and the "true" slant delay obtained from the independent receiver network. Computed correction error is based on the GIVE algorithm applied to the NSTB data. When a point in this plot lies above the diagonal line, the GIVE algorithm has bounded the true error. Computed errors at 10 meters or above appear at the top of the plot.

A successful GIVE algorithm must satisfy the following two criteria: the correction errors derived from GIVE must be larger than ~99.9% of the actual correction errors; but the GIVE must not be so large as to unnecessarily restrict the availability of high-accuracy navigation. For both days presented here, GIVE adequately bounds the user errors. On August 6th, errors are not bounded for only 7 points of a total of 13494. These points are plotted in Figure 5 showing that the underestimation is quite small. On August 25th, the true error exceeded computed for only 11 out of 11,815 processed points. For these 11 outliers, the GIVE underestimated the true error by at most 30 cm.

To assess whether the GIVE algorithm is too conservative, refer again to the scatter plots in Figures 3 and 4. These show the fraction of errors below a given level (to the right and top of the figure). For example, on the quiet day (Figure 3) the fraction of true errors below 2 meters is 100%. The fraction of computed errors below 2 meters is a fairly high 91.3%. A number close to 100% is desirable. According to Raytheon estimates [Peck, 1998], precision approach should be generally available if roughly 90% or more of the GIVE values fall below 2 meters. Therefore, this GIVE algorithm is probably not overly conservative. Comparing these two days, it is interesting to note that during the moderate storm (Figure 4), the percentage of computed errors below 2 meters is actually somewhat larger, and the number of times the errors were underestimated is similar (~0.1%). This suggests that availability may remain high during most storms without harming integrity. Of course, further analysis is necessary. The same conclusion cannot be reached for severe storms, as we discuss next.

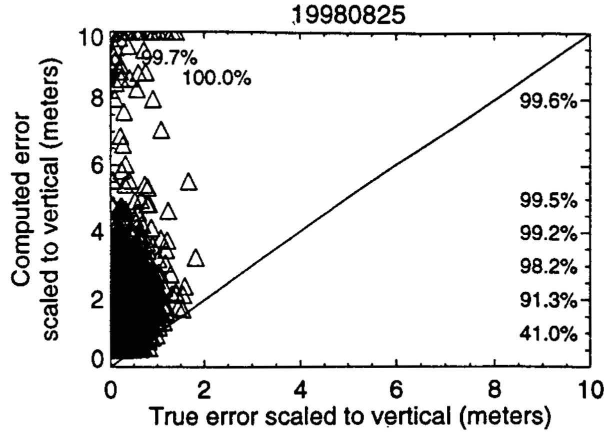


**Figure 3**. GIVE validation results for nominal ionospheric conditions on August 25, 1998.

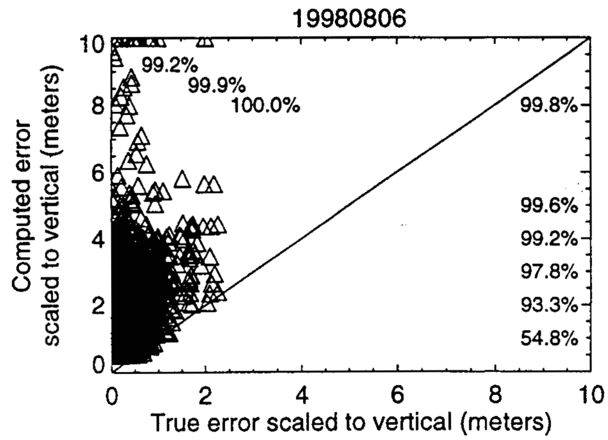


**Figure 4**. GIVE results for a moderately disturbed day (August 6, 1998; global geomagnetic index Ap=69).

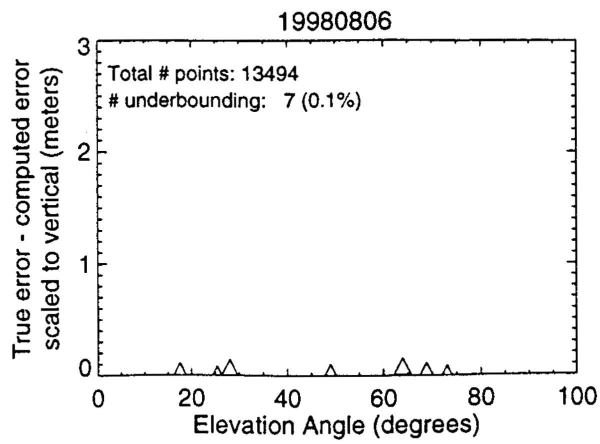


**Figure 5**. Detailed view of the cases where the computed error was less than the true error.

## 4.1 SEVERE STORM PERFORMANCE

It is important to assess the new GIVE algorithm under as wide a variety of ionospheric conditions as possible. Fortunately, a very intense geomagnetic storm occurred on August 27 when the geomagnetic Ap index exceeded 100. Over one year, storms of such intensity occur roughly 0.4% percent of the time somewhere on Earth, and so will probably affect WAAS perhaps a few times per year (Chavin, 1996b).

The algorithm performance during the disturbance is shown in Figures 6 and 7. The percentage of true errors above 2 meters remains quite low (0.6% = 1 – 99.4%). The fraction of computed errors above 2 meters is increased under storm conditions (to 16.6% = 1 – 83.4%). This may result in decreased availability of precision approach which is not surprising for severe disturbances. The most significant new feature is the increased number of points (0.7%) where the computed error did not bound the true error. On rare occasions, the magnitude of the underbounding exceeded one meter. These results are qualitatively different than those observed for the more moderate conditions.

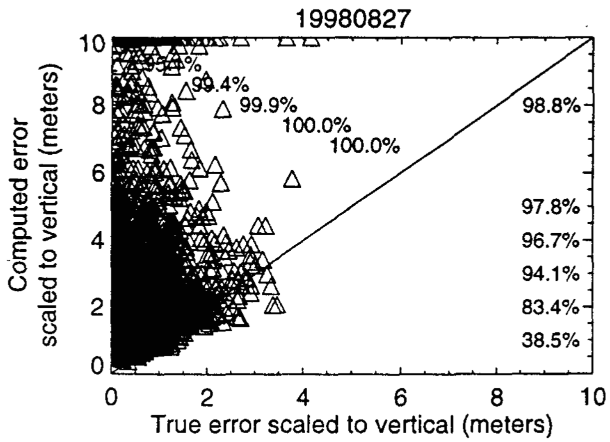


**Figure 6.** GIVE results for a severely disturbed day (Ap=112).

Detailed analysis revealed that significant underbounding occurs in the presence of a narrow but deep trough in the ionosphere that formed due to the disturbance. The width of the trough appeared to be below the resolution limit of the WAAS system [Mallis and Essex, 1993; Kersley *et al.*, 1997]. User lines-of-sight piercing the ionosphere in the trough region measure delays that are significantly below the corrections.

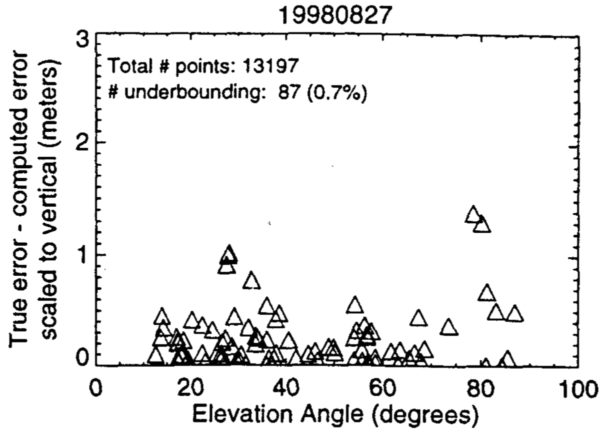


**Figure 7.** Detailed view of cases where GIVE led to an underestimation of the error during a severe storm.

The GIVE is based on data and correction values obtained from the four quadrants surrounding an IGP. Very narrow disturbance features cause too few outliers to significantly affect the goodness-of-fit, and so GIVE is relatively insensitive to such features. We are initiating a careful study to determine the best way of extending the GIVE formulation to cover theses types of events. Nevertheless, we are encouraged that even in this very severe storm, the number of outliers is quite small and are rarely larger than one meter (Figure 7).

## 5. CONCLUSIONS

Three contributions to the user's correction uncertainty are modeled by the GIVE algorithm: 1) uncertainty in the vertical grid delay; 2) unmodeled spatial decorrelation of the ionosphere; and 3) errors converting the vertical corrections to slant. An important feature of the algorithm is that it depends on the ionospheric conditions local to an IGP. In general, the relative contributions of these error terms will depend on geographic location within the US. For example, under normal ionospheric conditions, horizontal delay gradients in the ionosphere increase towards the south. Thus, the GIVE model predicts that users in the southern US generally incur larger errors converting vertical delays to slant than users in the north.

The next step in the development of the algorithm is to carefully verify the error model implicitly contained in the three GIVE terms. A more accurate error model will increase the availability of precision navigation because there will be less need to rely on overly large safety factors to ensure integrity.

The validation system we have developed (Figure 1) is being prepared for continuous daily operation. We will use the daily data to measure the various error contributions that are being modeled. Although it is not always possible to cleanly separate the different components of error, general trends can be distinguished. These results will be reported in a future paper.

Refinement of the GIVE terms is important because Raytheon plans to use the GIVE formulas in an independent safety monitor that runs concurrently with GIVE. A user ionosphere vertical error (UIVE) algorithm will continuously assess the worst-case user vertical delay uncertainty near a given IGP. If a user's correction uncertainty becomes too large, the surrounding IGPs may be declared unusable. It is important that realistic models of user error be used for this strategy.

The validation process cannot be considered complete until the GIVE algorithms are incorporated into the actual WAAS system. The formal error term (equation 1) is sensitive to the noise characteristics of the receivers since it depends on the residual differences between delay measurements and the fit. The spatial decorrelation and gradient terms depend to some extent on the precise tuning of the delay estimation filter.

We are continuing development of the algorithm to improve performance in the presence of ionospheric structures that are smaller than the resolution of the WAAS correction algorithm[2]. Such features may develop during severe ionospheric disturbances in the form of deep troughs. We are currently analyzing the distribution of residuals (measured minus computed delay) in the vicinity of the troughs. A particular distribution of outliers may signal the presence of these extreme conditions, and the GIVE can be increased accordingly. This may be preferable to simply increasing the factor $\alpha$ associated with the formal error term, which may adversely affect availability even when conditions are not too severe. The impact of these disturbance features on user positioning accuracy requires further investigation.

## ACKNOWLEDGMENTS

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## NOTES

1. If the users must lie within the boundaries of CONUS, the maximum distance might be less than if location anywhere in a quadrant is assumed. The GIVE algorithm has "hooks" so that the border effects can be considered.

2. We are considering here structures that are below the 5 degree resolution limit of the WAAS grid, but not structures so small that they cause scintillation effects. We expect that in the presence of scintillation, the GIVE formulation will require some modification and we will analyze these cases separately.

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